Grade 6 Math Circles

November 7/8/9, 2023

Inequalities and Absolute Values - Solutions

Exercise Solutions

Exercise 1

Fill in the blanks with a $<$, $>$, or a number so each statement is true.

(a) 4 ____ -2

(b) $0 >$ ____

(c) ____ < -10

(d) 150 ____ -150

(e) -5 ____ 0

(f) ____ $>$ ____ > -13

Exercise 1 Solution

(a) $4 > -2$

(b) $0 > -5$

(c) $-12 < -10$

(d) $150 > -150$

(e) $-5 < 0$

(f) $10 > 9 > -13$

Exercise 2

Solve the following equations.

(a) $2x - 7 = 23$

(b) $4y - y + 1 = 22$

(c) $4z + 1 = -5 + z$

Solve this system of equations.

(d) $6a - 1 = 29 + a$

AND

$2a - b = 16$

Exercise 2 Solution

(a) $2x - 7 = 23$

$2x = 30$

$x = 15$

(b) $4y - y + 1 = 22$

$3y + 1 = 22$

$3y = 21$

$y = 7$

(c) $4x + 1 = -5 + x$

$3x = -6$

$x = -2$

(d) $6a - 1 = 29 + a$

$5a = 30$

$a = 6$

$2a - b = 16$

$2(6) - b = 16$

$12 - b = 16$

$b = -4$

Exercise 3

Solve the following inequalities.

(a) $x + 2 > 11$ (b) $9 - 3a - a < 1$ (c) $8 + 2b > 4b + 2$

Exercise 3 Solution

(a)

$$\begin{aligned} x + 2 &> 11 \\ x &> 9 \qquad \text{(subtract 2 from both sides)} \end{aligned}$$

(b)

$$\begin{aligned} 9 - 3a - a &< 1 \\ 9 - 4a &< 1 \qquad \text{(combine like terms)} \\ -4a &< -8 \qquad \text{(subtract 9 from both sides)} \\ a &> 2 \qquad \text{(divide both sides by } -4\text{, flip the sign)} \end{aligned}$$

(c)

$$\begin{aligned} 8 + 2b &> 4b + 2 \\ 8 - 2b &> 2 && \text{(subtract } 4b \text{ from both sides} \\ -2b &> -6 && \text{(subtract 8 from both sides)} \\ b &< 3 && \text{(divide both sides by } -2\text{, flip the sign)} \end{aligned}$$

Exercise 4

Evaluate the following expressions. Ensure you follow the correct order of operations.

(a) $|5 - 3| + |3 - 5|$ (b) $|6 \times (-4)| + |-16 \div 2|$ (c) $-|2 + (-7)|$

Exercise 4 Solution

(a) $|5 - 3| + |3 - 5|$

$= |2| + |-2|$

$= 2 + 2$

$= 4$

(b) $|6 \times (-4)| + |-16 \div 2|$

$= |-24| + |-8|$

$= 24 + 8$

$= 32$

(c) $-|2 + (-7)|$

$= -|-5|$

$= -5$

Exercise 5

Determine the values of x such that the equations are true.

(a) $|x + 5| = 9$ (b) $|x - 3| = 7$

Exercise 5 Solution

(a) $|x + 5| = 9$ asks us to find the numbers such that the distance from x to -5 is nine. 4 is nine away from -5 and -14 is nine away from -5. Therefore $x = 4$ or $x = -14$.

(b) $|x - 3| = 7$ asks us to find the numbers such that the distance from x to 3 is seven. 10 is seven away from 3 and -4 is seven away from 3. Therefore, $x = 10$ or $x = -4$.

Exercise 6

It's recommended that you use a number line to help with these questions.

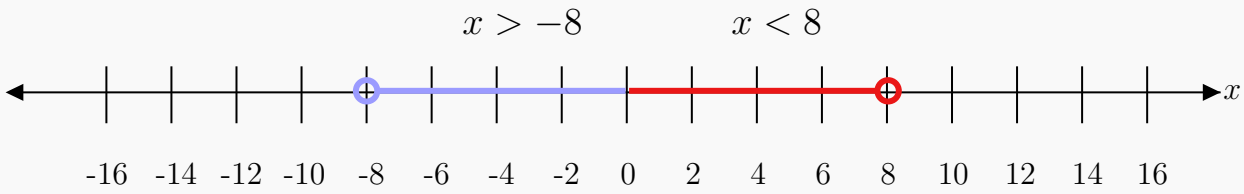
(a) Determine all values of x such that $|x| < 8$.

(b) Determine all values of x such that $|x + 4| < 8$.

(c) Determine all values of x such that $|x - 2| > 3$.

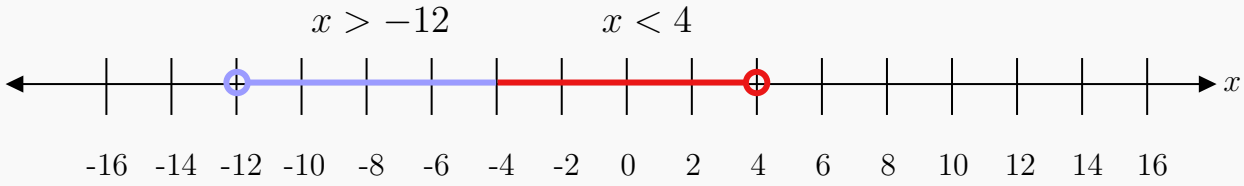
Exercise 6 Solution

(a) This asks us to find the numbers that are less than eight away from 0. The number line below shows this.



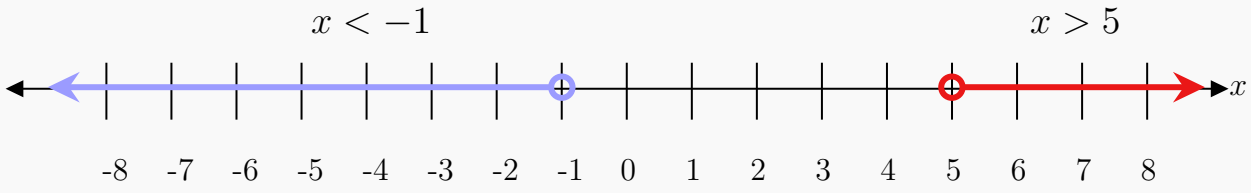
Anything outside this range would be too far away from 0. Therefore, $-8 < x < 8$ is the range of x values satisfying the inequality.

(b) This asks us to find the numbers that are less than eight away from -4. The number line shows this.



Anything outside this range would be too far away from -4. Therefore, $-12 < x < 4$ is the range of x values satisfying the inequality.

(c) This asks us to find the numbers that are greater than three away from 2. The number line shows this.



Anything outside this range would be too close to 2. Therefore, $x < -1$ **or** $x > 5$ is the range of x values satisfying the inequality.

Problem Set Solutions

1. Place a $<$, $>$, or a number in each blank to make the inequality true.

(a) 14 _____ 20
(b) $|-14| >$ _____
(c) $|-19|$ _____ $|-3|$
(d) $|2 \times (-4)| <$ _____
(e) $2-5$ _____ $|2-5|$
(f) $|x|$ _____ -1

Solution:

(a) $14 < 20$
(b) $|-14| > 13.9$
(c) $|-19| > |-3|$
(d) $|2 \times (-4)| < 10$
(e) $2-5 < |2-5|$
(f) $|x| > -1$

2. The city is planning the finances of the buildings they need to construct. They know:

   - Ice rinks are more expensive than apartments.
   - Garages are less expensive than houses.
   - Houses are less expensive than apartments.

   Write a single compound inequality that lists the cost of constructing the buildings from least to greatest.

   Solution: Start by writing down exactly what we are told.

   Ice Rink $>$ Apartment Garage $<$ House House $<$ Apartment

   Since we want to list them from shortest to tallest, flip any $>$ signs to $<$ signs and see if we can combine any inequalities.

   Apartment $<$ Ice Rink Garage $<$ House House $<$ Apartment

   Notice how the House is in between the Garage and the Apartment. So we can combine these two into a compound inequality.

   Apartment $<$ Ice Rink Garage $<$ House $<$ Apartment

   Notice how the Apartment is in between the House and the Ice Rink. So we can combine these two into a compound inequality.
   The final compound inequality is Doghouse $<$ House $<$ Apartment $<$ Ice Rink.

3. Evaluate the following expressions to a single integer or fraction. Ensure you follow the correct order of operations.

(a) $\dfrac{3 \times (2+4)}{3-1}$

(b) $2|1 - 4 \times 3|$

(c) $-3 \div |-6 - 3 \times 3|$

(d) $|10 \times (-2)(3)| \div (-|3-6|)$

Solution:

(a)
$$\begin{aligned}\frac{3 \times (2+4)}{3-1} &= \frac{3 \times (6)}{2} \\ &= \frac{18}{2} \\ &= 9\end{aligned}$$

(b)
$$\begin{aligned}2|1 - 4 \times 3| &= 2|1 - 12| \\ &= 2|-11| \\ &= 2(11) \\ &= 22\end{aligned}$$

(c)
$$\begin{aligned}-3 \div |-6 - 3 \times 3| &= -3 \div |-6 - 9| \\ &= -3 \div |-15| \\ &= -3 \div 15 \\ &= -\tfrac{1}{5}\end{aligned}$$

(d)
$$\begin{aligned}|10 \times (-2)(3)| \div (-|3-6|) &= |10 \times (-6)| \div (-|-3|) \\ &= |-60| \div (-3) \\ &= 60 \div (-3) \\ &= -20\end{aligned}$$

4. Determine all values of x in the following mix of equations and inequalities.

(a) $2x + 10 = -2$
(b) $2 - x > 1$
(c) $\frac{9}{2}x - \frac{5}{2}x - 7 = 1$
(d) $|(-4)(-2)(x) \div 8| = 48$
(e) $|x + 1| < 5$
(f) $|x - 2| > 6$

Solution:

(a) $2x + 10 = -2$

$2x = -12$

$x = -6$

(b) $2 - x > 1$

$-x > -1$

$x < 1$

(c) $\frac{9}{2}x - \frac{5}{2}x - 7 = 1$

$2x - 7 = 1$

$2x = 8$

$x = 4$

(d) $|(-4)(-2)(x) \div 8| = 48$

$|8x \div 8| = 48$

$|x| = 48$

$x = 48$ **or** $x = -48$

(e) $|x + 1| < 5$ asks to find numbers that are less than five away from -1.

-6 is exactly five away from -1 and 4 is exactly five away from -1. Therefore, any numbers closer to -1 than these numbers solve this inequality. In other words, $x > -6$ **and** $x < 4$ which can be combined into $-6 < x < 4$.

(f) $|x - 8| > 6$ asks to find the numbers that are greater than six away from 8.

2 is exactly six away from 8 and 14 is exactly six away from 8. Therefore, any numbers further away from six than these numbers solve the inequality. In other words, $x < 2$ **or** $x > 14$.

5. (a) Try you find a value for x such that $|x - 2| = -1$?
If you find a number, substitute it back into the orignal equation to check if it is correct.
If you cannot find a number, explain why!

(b) Find all values of x such that $|x - 2| > -1$.

Solution:

(a) There are no solutions to this equation! This is because the absolute value *always* produces a non-negative number (non-negative meaning 0 or positive). Therefore, it is impossible for the absolute value of *any number* to be equal to -1. So, there are

no solutions.

Another way you could answer this follows the intuition we have used during this lesson. This equation is asking us to find all numbers that are -1 away from 2. But distance is always greater than or equal to 0, so this question doesn't make sense. You can't be -1 away from something. Therefore, there are no solutions.

(b) Since the absolute value is *always* greater than zero or equal to zero, $|x-2| > -1$ is true *for all* x-values!

6. Complete the following HANGMAN activity that tests most of the skills you learned today!

WHAT IS NEXT WEEK'S TOPIC?

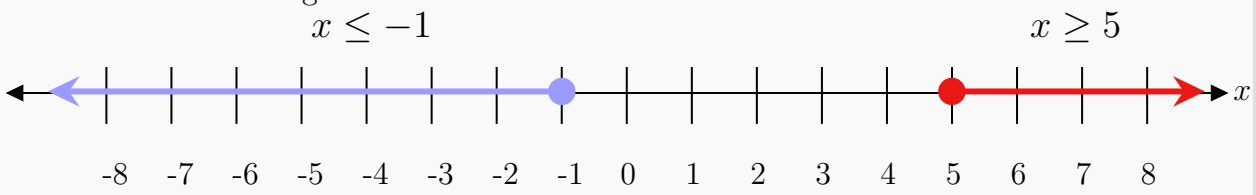
$x > 10$	No Solutions	$x > 5$ OR $x < -3$	$x = 5$ OR $x = -13$	$x \geq -28$	$x = -\frac{1}{2}$	$x = 3$	$x > -9$ AND $x < -5$	$x < \frac{7}{5}$	No Solutions

A $5x - 3x + 1 = 2$	B $x + 5 > 2x + 1$	C $\|x - 1\| > 4$	E $x + 7 < 2x - 3$	I $\|x + 7\| < 2$
L $x = \|2 + 7 \times 6\| - 1$	M $3x - 5 < 5x + 1$	N $\|2x + 1\| = -4$	O $-3.5x + 7 > 1.5x$	P $2x + 1 = 0$
R $\|x + 4\| = 9$	T $2x = \|4 - 10\|$	U $2x + 7 = 2$	Y $9 \times 3 + 2x \geq x - 1$	Z $x^2 - 1 = 0$

Solution: ENCRYPTION is next week's topic! Hangman is a type of encryption!

7. What do you think the symbols $\leq$ and $\geq$ mean?

Solution: $\leq$ is the "less than or equal to symbol" and $\geq$ is the "greater than or equal to symbol." These are called **non-strict** inequalities and on a number line, we fill in the circle because the endpoint is included in the interval. For example, $x \leq -2$ **or** $x \geq 5$ looks like the following.



8. Combine the following inequalities with an "and" or an "or".

(a) $x < 3$, $x > 5$ (b) $x > -4$, $x < 10$ (c) $x > 7$, $x > -3$, $x < 4$

Solution:

(a) $x < 3$ **or** $x > 5$

(b) $x > -4$ **and** $x < 10$ $(-4 < x < 10)$

(c) $x > -3$ **and** $x < 4$ **or** $x > 7$ $(-3 < x < 4$ **or** $x > 7)$

9. CHALLENGE QUESTION

The goal for this question is to solve an inequality with multiplication inside the absolute value, instead of just addition or subtraction.

(a) Remember that both $|9| = 9$ and $|-9| = 9$. Use this to determine all values of x such that $|2x - 2| = 9$ and label them on a number line.

(b) Determine all values of x such that $|2x - 2| < 9$ and label them on a number line. Part (a) should help with this.

(c) Use the same steps/ideas from part (a) and (b) to determine all values of x such that $|4x + 8| > 12$.

Solution:

(a) Since both $|9| = 9$ and $|-9| = 9$, there are two ways for $|2x - 2| = 9$ to be true. Either:

- $2x - 2 = 9$ (equation 1) or;
- $2x - 2 = -9$ (equation 2)

We need to solve both of these equations.

$2x - 2 = 9$

$2x = 11$

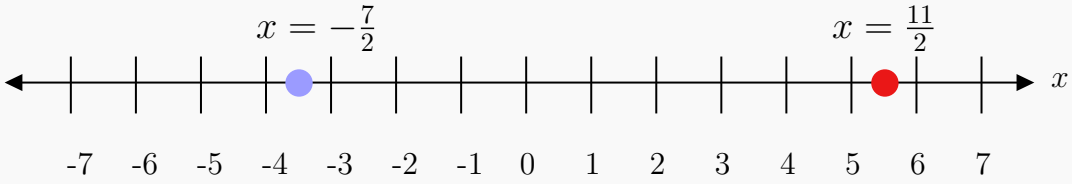
$x = \frac{11}{2}$

$2x - 2 = -9$

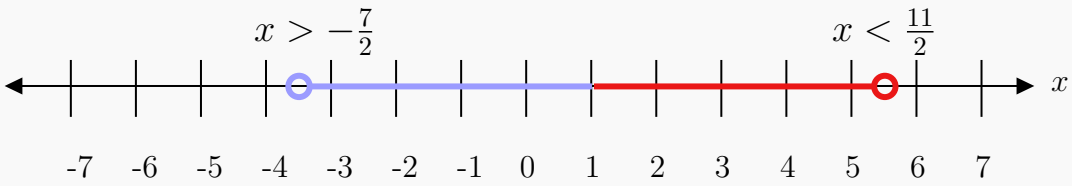
$2x = -7$

$x = -\frac{7}{2}$

Therefore, $x = \frac{11}{2}$ **or** $x = -\frac{7}{2}$.



(b) In part (a), we determined that $x = \frac{11}{2}$ **or** $x = -\frac{7}{2}$ are solutions to $|2x-2| = 9$. Now we need to find x values such that $|2x - 2| < 9$. This is easy with the help of the number line from part (a). x is just all number between the points on the number line!



Therefore, $x > -\frac{7}{2}$ **and** $x < \frac{11}{2}$, which can also be written $-\frac{7}{2} < x < \frac{11}{2}$.

(c) First, try to solve $|4x + 8| = 12$. This means we need to solve two equations, similar to part (a).

$4x + 8 = 12.$

$4x = 4$

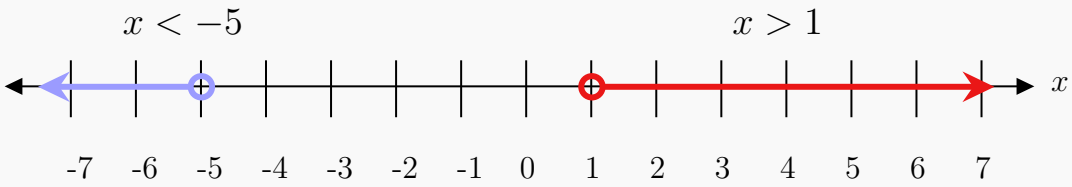
$x = 1$

$4x + 8 = -12.$

$4x = -20.$

$x = -5$

Since the equation we are actually asked to solve is $|4x + 8| > 12$, we need numbers that are outside of these numbers.



Therefore, $x < -5$ **or** $x > 1$.